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Large amplitude plasma wave excitation by means of sequences of short laser pulses

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The generation of a large amplitude electron plasma wave by means of n short laser pulses is studied in the *quasistatic approximation* using a one-dimensional relativistic cold plasma model. The problem is reduced to considering the coupled equations for the vector and the scalar potentials. Preliminary results are obtained by means of a linear stability analysis of the system and by the integration of the Poisson equation for constant pulses. The energetics and time evolution of the excitation process are described by the numerical integration of the relevant equations.

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The possibility of exciting large amplitude electron plasma waves, either by means of pulses of electromagnetic (e.m.) radiation [1] or by injection of bunches of relativistic electrons [2], has drawn much attention after various proposals of using strong electrostatic (e.s.) potential gradients to accelerate charged particles to TeV energies in relatively short distances (tens of meters) [1,3]. Moreover, the recent technological availability of very intense laser-pulse sources [4–6] has been accompanied by several theoretical studies which have reconsidered in much detail the mechanism of wave excitation [laser wake-field accelerator (LWFA)] based on the strongly nonlinear interaction between a short ($\tau_L \cong 0.1-1$ ps) e.m. pulse and plasma electrons, via the relativistic ponderomotive force associated with the high frequency field [7–17]. Both analytical and numerical analysis have shown that the LWFA scheme seems to offer some advantages if compared with other methods like plasma beat wave accelerator (PBWA) [18–25] or plasma wake-field accelerator (PWFA) [26–28]. The LWFA scheme, however, requires very intense pulses if a sufficiently large plasma wave is to be actually generated.

In this paper we study the possibility of using a sequence of n short, medium intensity (i.e., $A \leq 1$, where $A \rightarrow eA/m_e c^2$ is the dimensionless amplitude of the vector potential) e.m. pulses as a driver for large amplitude plasma waves. Such a choice seems to offer some advantages with respect to the use of a single, high amplitude ($A \gg 1$) pulse, since medium intensity pulses (a) are expected to be more stable to envelope perturbations, (b) are already available for experiments ($A \approx 1$ corresponds to a laser intensity $I \approx 1.38 \times 10^{18}$ W/cm^2, for a wavelength $\lambda \approx 1$ μm).

The propagation of laser pulses and the excitation of e.s. oscillations in the plasma are described on the basis of a cold relativistic fluid plasma model in which collisions and ion dynamics are neglected. Moreover, a one-dimensional description is used, that is, a uniform system in (x,y) planes perpendicular to the (z) axis along which the pulses propagate is considered. The assumption of "slab" pulses is well satisfied in the case of lasers based on the chirped pulse amplification concept [4].

The variables $\xi = z - v_\phi t$ and $\tau = t$ are introduced, where v_ϕ is the phase velocity of the excited plasma wave, approximately equal to $v_g = c(1-\omega_p^2/\omega_0^2)^{1/2}$, the group velocity in the plasma of the e.m. pulse of frequency ω_0. $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency and m_e is the rest mass of the electron. Since it is assumed that $\omega_0 \gg \omega_p$, v_ϕ is slightly smaller than the speed of light.

If the duration of the interaction between the plasma and the pulses is smaller than the characteristic time of modification of the pulses due to nonlinear self-effects, the *quasistatic approximation* [14] can be introduced to simplify the fluid (continuity and momentum) equations. This corresponds to neglecting a direct dependence of the fluid variables on τ and assuming that they follow adiabatically the evolution of the e.s. potential of the plasma wave, $\phi(\xi,\tau)$, and the vector potential associated with the laser pulse, $\mathbf{A}_\perp(\xi,\tau)$. The problem may thus be formulated in terms of $\phi(\xi,\tau)$ and $\mathbf{A}_\perp(\xi,\tau)$.

Let us consider a circularly polarized e.m. pulse,

$$\mathbf{A}_\perp(z,t) = \tfrac{1}{2}[A_0(z,t)e^{i(k_0 z - \omega_0 t)} + \text{c.c.}](\mathbf{e}_x \pm i\mathbf{e}_y) , \quad (1)$$

where A_0 is the scalar complex amplitude of the vector potential.

From the set of Maxwell's and fluid equations we find the following system of differential equations [15–17]:

$$\frac{\partial^2 \phi}{\partial \xi^2} = \gamma_\phi^2 \left[\frac{v_\phi(1+\phi)}{[(1+\phi)^2 - \gamma_\perp^2/\gamma_\phi^2]^{1/2}} - 1 \right] , \quad (2)$$

$$\frac{1}{\gamma_\phi^2}\frac{\partial^2 A_0}{\partial \xi^2} + 2v_\phi \frac{\partial^2 A_0}{\partial \xi \partial \tau} - \frac{\partial^2 A_0}{\partial \tau^2} + i2\omega_0 \frac{\partial A_0}{\partial \tau} = \left[-1 + \frac{v_\phi}{[(1+\phi)^2 - \gamma_\perp^2/\gamma_\phi^2]^{1/2}} \right] A_0 , \quad (3)$$

where $\gamma_\phi = 1/(1-v_\phi^2)^{1/2}$, $\gamma_\perp = (1+|A_0|^2)^{1/2}$, and the following dimensionless variables are used:

$$\omega_p \tau \rightarrow \tau, \quad \frac{\omega_p}{c}\xi \rightarrow \xi, \quad \frac{v}{c} \rightarrow v, \quad \frac{e\mathbf{A}}{mc^2} \rightarrow \mathbf{A}, \quad \frac{e\phi}{mc^2} \rightarrow \phi .$$

(a) Linear stability analysis. Typical time and space scales on which external perturbations to the envelope

$A_0(\xi,\tau)$ can become unstable, as well as the dependence of such an instability on the amplitude and propagation velocity of the e.m. radiation in the plasma, may be estimated from a linear stability analysis of Eqs. (2) and (3). For the sake of simplicity we introduce the new variables

$$\bar{x}=\gamma_\phi\xi,\quad \bar{t}=\frac{\tau}{2v_\phi\gamma_\phi},\quad \Psi=1+\phi\ , \tag{4}$$

and consider the unperturbed solution of the system in the form $A_0(\bar{x},\bar{t})=\bar{A}\exp(i\beta\bar{x})$, $\Psi(\bar{x},\bar{t})=\bar{\Psi}=\gamma_\perp$, where $\beta=-\sqrt{1-1/\gamma_\perp}$, $\gamma_\perp=(1+\bar{A}^2)^{1/2}$, which corresponds to a constant e.s. potential and a spatially modulated vector potential of amplitude $\bar{A}$. We notice that, because of the definition of ξ and $\bar{x}$, v_ϕ is the velocity of propagation of this modulation in the plasma. We analyze the evolution of small perturbations by posing:

$$\begin{aligned}&A_0(\bar{x},\bar{t})=[\bar{A}+a(\bar{x},\bar{t})]e^{i\beta\bar{x}},\quad |a|\ll|\bar{A}|\\ &\Psi(\bar{x},\bar{t})=\bar{\Psi}+\varphi(\bar{x},\bar{t}),\quad |\varphi|\ll|\bar{\Psi}|\end{aligned} \tag{5}$$

and linearizing Eqs. (2) and (3) in the small perturbations $a(\bar{x},\bar{t})$ and $\varphi(\bar{x},\bar{t})$. By introducing an expansion in normal modes of $\varphi(\bar{x},\bar{t})$ and of the real and imaginary parts of $a(\bar{x},\bar{t})$, i.e., $a_R,a_I,\psi\approx\exp[i(X\bar{x}-\Omega\bar{t})]$, the following sixth degree in X dispersion relation is derived:

$$\left[X^2-\frac{1}{v_\phi^2\gamma_\phi^2\gamma_\perp}\right]\left\{X^2[X-\Omega]^2-\left[2\beta X-\left[\beta+\frac{1}{v_\phi}\right]\Omega\right]^2\right\}+\frac{\bar{A}^2}{v_\phi^2\gamma_\phi^2\gamma_\perp^3}X(X-\Omega)\left[X^2+\frac{1}{\gamma_\perp}\right]=0\ . \tag{6}$$

Due to the quasistatic approximation, Eq. (6) is valid only for perturbations of frequency much smaller than the plasma frequency (in the drifting reference system in which the laser beam is quasistationary), which means that $|\Omega|\ll 2v_\phi\gamma_\phi$ [see Eq. (4)].

Two classes of complex solutions of Eq. (6) exist: a first one, characterized by higher values of $|\mathrm{Im}X|$, corresponds to evanescent perturbations already present for $\bar{A}=0$; a second one, with smaller values of $|\mathrm{Im}X|$, is instead characteristic of nonzero e.m. fields. Perturbations belonging to this second class, which we interpret as envelope instabilities of the modulated e.m. wave beam coupled with relativistic electron plasma oscillations, may grow exponentially in amplitude and thus lead the system to instability.

To perform an analysis of the dependence of such instability on the values of v_ϕ and $\bar{A}$ we go back to the variables ξ,τ, that is, we consider perturbations of the form $\exp[i(X'\xi-\Omega'\tau)]$, where $X'=\gamma_\phi X$, $\Omega'=\Omega/(2v_\phi\gamma_\phi)$. We can characterize the degree of the instability of the envelope by means of the width $\Delta\Omega'$ of the frequency interval which may be unstable and the maximum growth rate $\Gamma'=\max|\mathrm{Im}X'|$. Moreover, an estimate of the characteristic time τ_E of the instability can be obtained from the relation $\tau_E=1/(\Gamma' v_g')$, where v_g' is the group velocity of a superposition of unstable modes.

By solving Eq. (6) and making use of the expression of v_g' in the limits $\Omega\rightarrow 0$, $X\rightarrow 0$, we obtain the plots of $\Delta\Omega'$ and τ_E vs $\bar{A}$ reported in Figs. 1 and 2. Figure 1 shows that, for a given v_ϕ value, the width of the unstable region $\Delta\Omega'$ grows with $\bar{A}$, while we see from Fig. 2 that the characteristic time of instability has a dependence $\tau_E\approx 1/\bar{A}^\alpha$ with $\alpha\approx 1$. We notice that, for not too large γ_ϕ values and $\bar{A}>1$, unstable perturbations of the pulse envelope can manifest themselves on time scales comparable to those of the plasma wave excitation. As expected, for a given γ_ϕ, envelope stability improves at $\bar{A}<1$ and one is encouraged to look for excitation mechanisms which are effective at limited $\bar{A}$ values.

(b) Plasma waves excited by a given sequence of laser pulses. Let us show that a large amplitude e.s. wave can be generated by properly injecting in the plasma a sequence of n medium intensity laser pulses. This would allow us to produce large electric fields with the laser sources already available, and to overcome the stability problems previously discussed.

To do this, we first consider a sequence of nonevolving rectangular pulses of constant amplitude A_0. Let L_i and d_i (with $i=1,\ldots,n$) be the length along the coordinate ξ of the ith pulse and the separation between the ith and the $(i+1)$th pulses, respectively. By solving Eq. (2), it

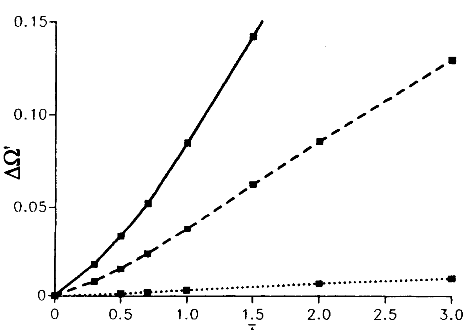

FIG. 1. $\Delta\Omega'$ vs $\bar{A}$ for $\gamma_\phi=7$ (full line), 10 (dashed line), 32 (dotted line). The dimensionless units defined in the text are used everywhere.

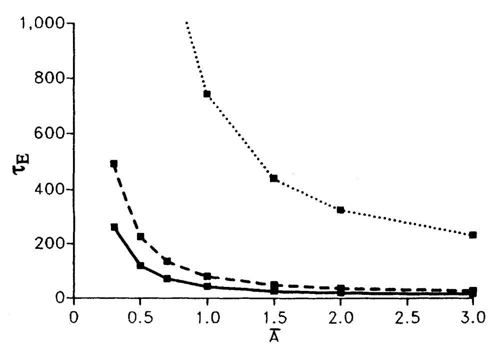

FIG. 2. τ_E vs $\bar{A}$ for $\gamma_\phi=7$ (full line), 10 (dashed line), 32 (dotted line).

turns out that the *optimal sequence* in generating plasma waves is that in which each pulse is switched on when the e.s. potential is minimum and is switched off at the subsequent potential maximum. $\phi(\xi)$ and $E(\xi)$ can be determined, in terms of elliptic integrals, up to the wavebreaking point, where the fluid description breaks down. As a result of this analysis, analytical expressions for L_i and d_i and, correspondingly, for the amplitude E_i of the plasma wave, in the wake of the ith pulse, have been obtained. In Fig. 3 these quantities are plotted as functions of the number of pulses (plots of d_i are omitted since they coincide with L_i ones, for $i>2-3$). The dashed lines show the values of L_i and E_i for $v_\phi=1$. In this case no wavebreaking can occur [17] and the electric field can increase indefinitely with n. Asymptotically, for $n\rightarrow\infty$, $E_n/E_{n-1}\propto\gamma_\perp$. On the contrary, when $v_\phi<1$, the maximum number of pulses allowed in order to avoid wavebreaking depends on A_0 and on v_ϕ itself. The curves referring to $\gamma_\phi=10$ (*a*), 32 (*b*), and 100 (*c*), for $A_0=0.7$ and 1, are reported. Each line ends at the maximum allowed n value for the given v_ϕ. In Fig. 3(b) the horizontal dotted lines indicate the wavebreaking amplitude of the field for each v_ϕ value considered. It is seen that, for a given A_0 value, an upper limit on the length of the *optimal sequence* exists due to the rapid increase of L_i with i.

(c) Self-consistent time evolution of the laser pulse-plasma system. To correctly describe the energetics of the e.s. wave excitation process the consistent evolution of the pulse envelope and frequency should be followed.

Equations (2) and (3) have been numerically integrated under the assumptions that $\omega_0\gg|\partial/\partial\tau|$ and $(1+\phi)^2\gg(1+|A_0|^2)/\gamma_\phi^2$, the latter corresponding to considering e.s. waves which do not undergo "wavebreaking" [12,17]. We notice that the time integration of the equation for $A_0(\xi,\tau)$ allows us to take the effects of the self-modifications of the laser pulses consistently into account. By writing the complex amplitude in the form

$$A_0(\xi,\tau)=|A_0(\xi,\tau)|\exp[iF(\xi,\tau)]\ ,$$

we can determine the "local" pulse frequency $\omega(\xi,\tau)=\omega_0+v_\phi\partial F/\partial\xi$. The following integral of motion exists:

$$I=\int_{-\infty}^{+\infty}d\xi[|A_0|^2+(v_\phi/\omega_0)(\partial F/\partial x)]$$
$$\approx\int_{-\infty}^{+\infty}d\xi|A_0|^2\omega(\xi,\tau)/\omega_0\ .$$

In Fig. 4, the process of e.s. field generation by means of $n=3$ pulses is shown for $\gamma_\phi=100$, at $\tau=2000$. At $\tau=0$ the pulses are almost rectangular with $A_0=1$ and suitable lengths and delays to maximize the excitation process. The corresponding electric field, $E=-\partial\phi/\partial\xi$, and frequency are also plotted. For the sake of comparison a single pulse of amplitude $A_0=2.6$, capable of generating a maximum electric field amplitude equal to that produced in the previous case, is also shown. With reference to an unperturbed uniform plasma with $n_e=10^{18}$ cm^{-3}, and to a laser wavelength $\lambda\approx0.3$ μm and a beam radius $r_0=30$ μm, it is interesting to compare the e.m. energies required in the two cases considered above. For $n=3$, the beam intensity is $I_3\approx1.5\times10^{19}$ W/cm^2, the time durations of the three pulses are $\tau_1\approx0.07$ ps, $\tau_2\approx0.08$ ps, $\tau_3\approx0.1$ ps, respectively, and the total energy in the three wave packets is $W_3\approx168$ J. In the case $n=1$, $I_1\approx10^{20}$ W/cm^2, $\tau_1\approx0.1$ ps, and $W_1\approx509$ J. At $\tau=0$, for $n=3$ the maximum electric field is $E\approx1.9$, and

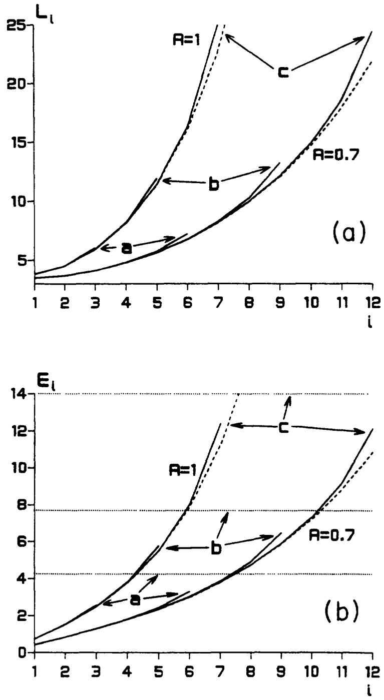

FIG. 3. The length L_i (a) of the ith laser pulse of the sequence and the corresponding maximum electric field generated, E_i (b), are plotted as functions of i for different v_ϕ values and $A_0=0.7$ and 1. The dashed lines refer to $v_\phi=1$ ($\gamma_\phi=\infty$). The full lines correspond to $\gamma_\phi=10$ (*a*), 32 (*b*), and 100 (*c*). The dotted horizontal lines indicate the wavebreaking field amplitudes for the v_ϕ values considered.

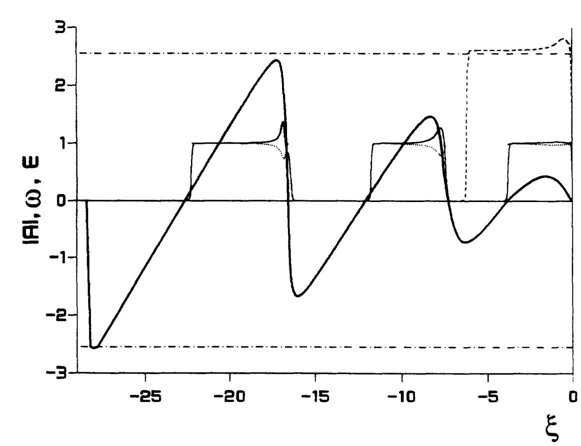

FIG. 4. The e.s. field (thick full line), the sequence of $n=3$ pulses (thin full lines) of initial amplitude $A_0=1$, and the e.m. radiation frequency $\omega(\xi,\tau)$ (dotted lines) vs ξ, are plotted at time $\tau=2000$, for $\gamma_\phi=100$. Dashed line shows the single laser pulse, of amplitude $A_0=2.6$, capable of generating the same electric field amplitude (horizontal dot-dashed lines).

for $n=1$ $E\approx 2.4$. At $\tau=2000$, in both cases $E\approx 2.6$, corresponding to an e.s. field amplitude of ≈ 260 GV/m. Therefore, besides confirming the potentialities of laser pulse sequences, the consistent analysis of the underlying nonlinear interaction demonstrates that this scenario allows one to lower the total energy demand on the laser system.

Before concluding, it should be mentioned that two-dimensional effects (see, for example, Refs. [7,29–33]) can, in principle, alter the physical picture that we have described here, introducing some limitations on the efficiency of the "resonant" plasma wave excitation process. However, it is possible to envisage physical situations in which on one side self-effects can avoid a transverse spreading of the laser beam, and, on the other side, typical time scales for spreading are longer than those required for an efficient pulse-plasma interaction. For example, for the parameters considered in Fig. 4, the dimensionless pulse diffraction time $\tau_f \approx Z_R$ [30], where $Z_R=\pi(r_{L0}^2/\lambda)(\omega_p/c)$ and r_{L0} are the Rayleigh length and the minimum spot size, respectively, is $\tau_f\approx 2000$, that is much larger than the time needed to excite the e.s. wave ($\tau_{ex}\approx$ a few units). Therefore, for sufficiently "flat" laser pulses, two-dimensional effects can be suitably limited by an appropriate choice of the experimental conditions.

In conclusion, we have shown that it is preferable and possible to generate large amplitude e.s. wake fields in a plasma by using sequences of medium intensity ($A\le 1$) laser pulses instead of a single large amplitude ($A>1$) e.m. wave packet, one major constraint being the availability of laser intensities up to $10^{18}-10^{19}$ W/cm^2. Moreover, the use of few pulses, i.e., $n\approx 3-4$, suitably dephased and of limited amplitude, allows us to decrease the growth rate of envelope instabilities and also to reduce the total energy required to get an electric field of a given amplitude. The required repetition rate of the source could be realized by using two or more synchronized lasers or, better, by suitably shaping a single long pulse. A more extensive analytical and numerical study supports the above discussed physical picture and will be the subject of a forthcoming extended paper.

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